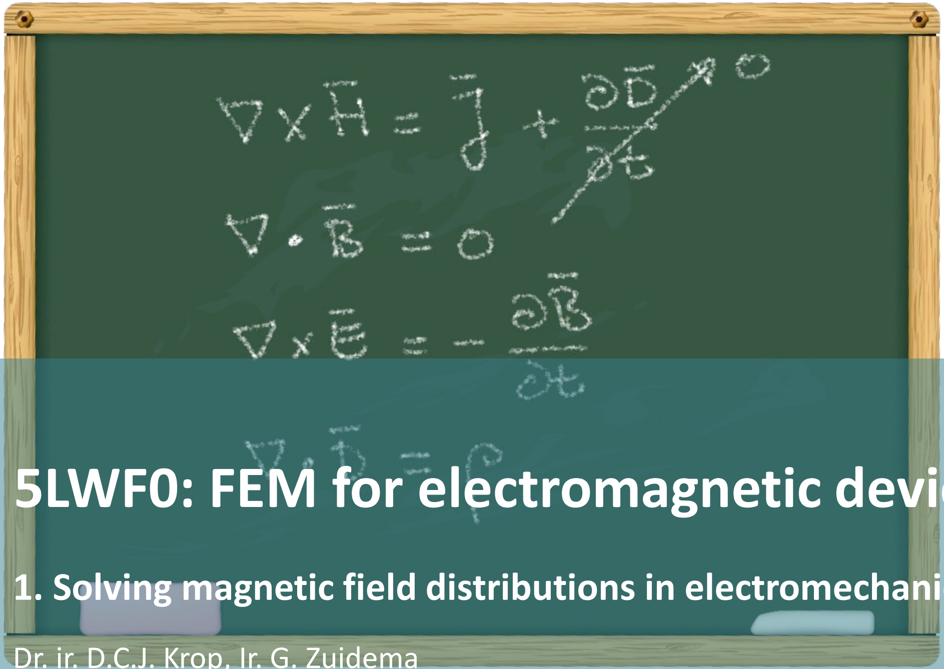
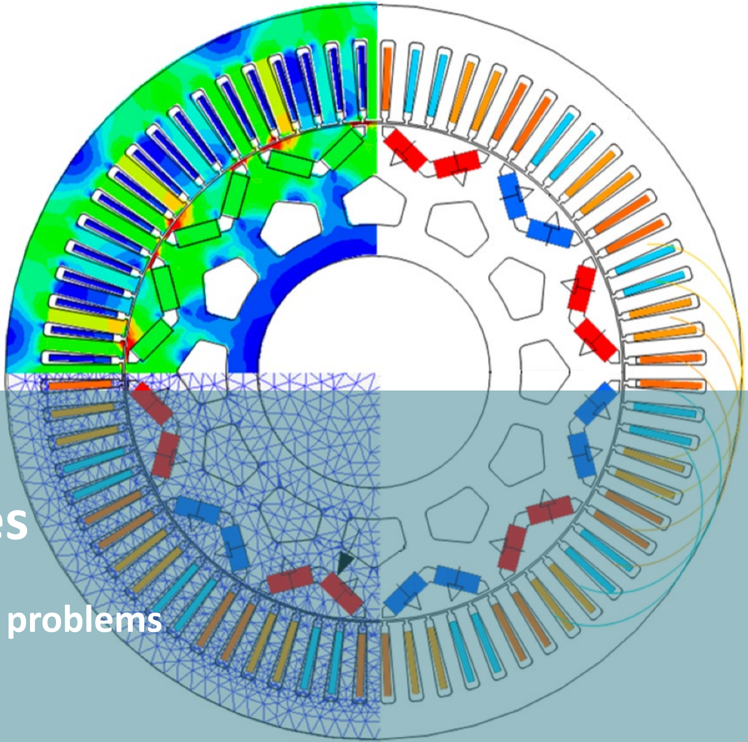




5LWF0: FEM for electromagnetic devices

1. Solving magnetic field distributions in electromechanical problems

Dr. ir. D.C.J. Krop, Ir. G. Zuidema

12 February 2025

Outline for the lecture of week 1

1. General information
   - Course material
   - Structure of the course
2. Recapitulation of Maxwell’s equations
   - Differential and integral form
   - Physical interpretation
3. Solving <u>B</u>oundary <u>V</u>alue <u>P</u>roblems (BVPs) analytically (1D)
   - Recapitulation of solving differential equations for a toroidal inductor
4. Numerical methods for solving BVPs (1D)
   - Method of weighted residuals
   - Solving a BVP numerically (1D)

General information

Contact information

Responsible lecturer:

Dr.ir. D.C.J. Krop
✉ d.c.j.krop@tue.nl FLX 2.089 ✆ 5716

Instructor:

Ir. G. Zuidema
✉ g.zuidema@tue.nl FLX 2.122 ✆ 3554

Required teaching material for 5LWF0

- Notebook

- Finite Element software package:
  - Altair Flux2D 2024.0.1
  - Download from internet
    - *https://altairone.com/home*
    - *Log on with your TU/e-email: A.B.lastname@student.tue.nl*
    - *Search for FLUX 2024 , download and install*
  - The software package FLUX is used under an **academic** license: **commercial use is strictly prohibited!**

  - The license server keeps logs of all FLUX activities!

Connection to other courses

Prior courses 5LWF0 builds on

- **5EWA0: Electromechanics**
  - **Magnetic Equivalent Circuits (MECs)**
  - DC-machines
  - Induction machines
  - **Synchronous machines**
- **5SWA0: Rotary PM machines**
  - **Brushless PM machines (AC and DC)**
  - **Winding configurations and saturation**
  - **EMF, MMF, flux linkage, inductance and torque production**
  - **dq0-axes decomposition**

5LWF0 follow-up

- Highly recommended for a graduation at the EPE group!

Objectives of 5LWF0

- Setting up a 2D electromagnetic problem in the Finite Element Method (FEM) software environment FLUX2D
- Discretization aspects, i.e. how to properly discretize (or mesh) the problem in order to obtain a reliable electromagnetic field distribution
- Using the ‘overlay’ feature in the FLUX2D FEM package for the analysis/design of electromagnetic devices with focus on BLPM machines
- Interpretation of the magnetic field distribution on the performance of a BLPM machine

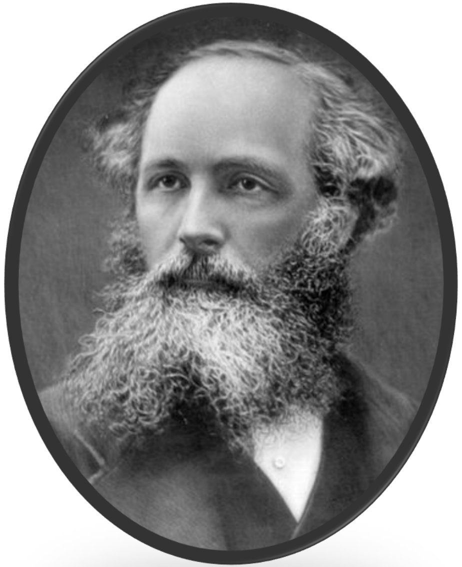
Exam

- Assignment :
  - Analysis of an electrical machine topology: 5 ECTS
  - Distributed to the students around the halfway-mark: week 4

- Written paper on your design approach and final design (60% , min. grade to pass: 5)
  - Max. 8 pages and stick to the predefined template
  - To be handed in 1 week prior to oral exam

- Oral exam (40%, min. grade to pass: 5)
  - Defend your design based on simulation results in combination with physical insight and design constraints
  - Time-slot based on student-ballot, but **not** later than 2 weeks after Q4 ends

Recapitulation of Maxwell’s equations



Recapitulation of Maxwell's equations

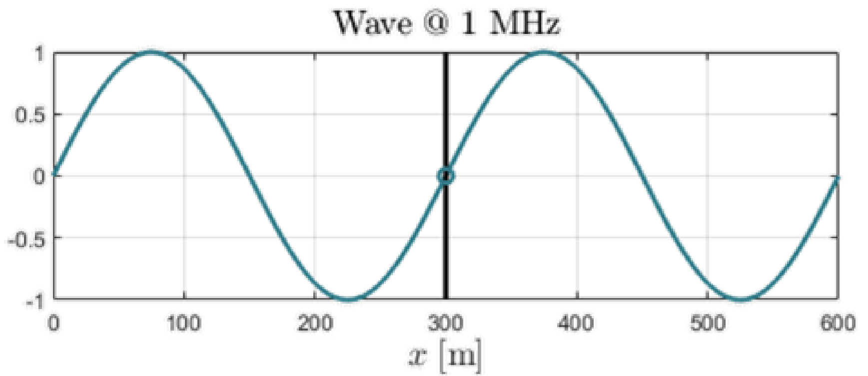
	Integral form	Differential form
Ampère's law	$\oint_{\partial\mathcal{S}} \vec{H}(\vec{r},t)\cdot\vec{\tau}\,\mathrm{d}\ell = \iint_{\mathcal{S}} \left[\vec{J}(\vec{r},t) + \frac{\mathrm{d}\vec{D}(\vec{r},t)}{\mathrm{d}t}\right]\cdot\vec{n}\,\mathrm{d}S$	$\nabla\times\vec{H}(\vec{r},t) = \vec{J}(\vec{r},t) + \frac{\partial\vec{D}(\vec{r},t)}{\partial t}$
Gauss's law (magnetism)	$\oiint_{\partial\mathcal{V}} \vec{B}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S = 0$	$\nabla\cdot\vec{B}(\vec{r},t) = 0$
Faraday's law	$\oint_{\partial\mathcal{S}} \vec{E}(\vec{r},t)\cdot\vec{\tau}\,\mathrm{d}\ell = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\mathcal{S}} \vec{B}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S$	$\nabla\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}(\vec{r},t)}{\partial t}$
Gauss's law	$\oiint_{\partial\mathcal{V}} \vec{D}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S = \iiint_{\mathcal{V}} \rho(\vec{r},t)\,\mathrm{d}V$	$\nabla\cdot\vec{D}(\vec{r},t) = \rho(\vec{r},t)$

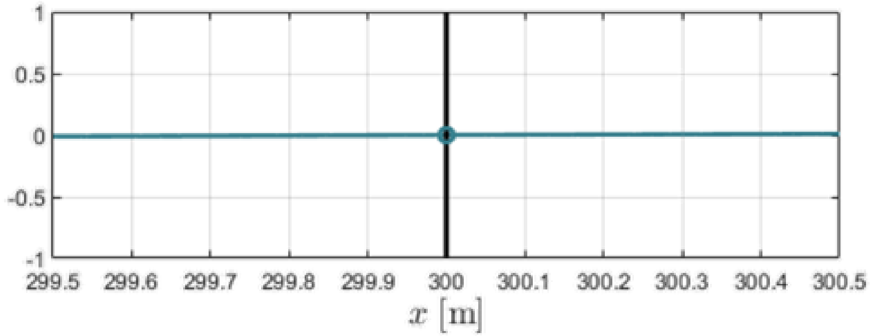
Classical electromagnetic problems are governed by four coupled **partial differential equations** (**PDE**): Maxwell's equations

Recapitulation of the quasi-static formulation

Quasi-static formulation for the majority of ***electromechanical*** problems

- For problems much smaller in size than the wavelength, the displacement term vanishes
- Time changes of field quantities within small domains can be considered instantaneous
- In electromechanical problems the free-charge, ρ, is often not of interest

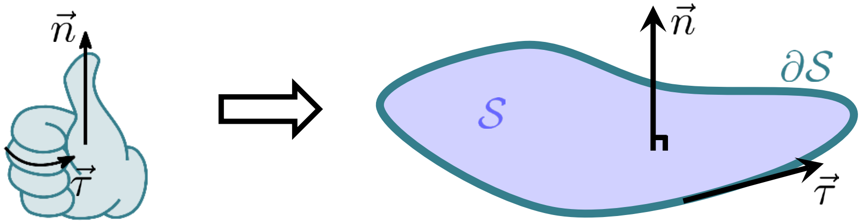




Recapitulation of the quasi-static formulation

	Integral form	Differential form
Ampère's law	$\oint_{\partial\mathcal{S}} \vec{H}(\vec{r},t)\cdot\vec{\tau}\,\mathrm{d}\ell = \iint_{\mathcal{S}} \vec{J}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S$	$\nabla\times\vec{H}(\vec{r},t) = \vec{J}(\vec{r},t)$
Gauss's law (magnetism)	$\oiint_{\partial\mathcal{V}} \vec{B}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S = 0$	$\nabla\cdot\vec{B}(\vec{r},t) = 0$
Faraday's law	$\oint_{\partial\mathcal{S}} \vec{E}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}\ell = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\mathcal{S}} \vec{B}(\vec{r},t)\cdot\vec{n}\,\mathrm{d}S$	$\nabla\times\vec{E}(\vec{r},t) = -\frac{\partial\vec{B}(\vec{r},t)}{\partial t}$

Relation between tangential, $\vec{\tau}$, and normal, $\vec{n}$, vectors given by the ***right***-hand rule:



Constitutive relations (material properties)

The four Partial Differential Equations (PDEs) are coupled via the electromagnetic material properties

For **isotropic**, **linear** materials the constitutive relations due to materials are

Magnetic

$$\vec{B}(\vec{r},t) = \mu_0 \mu_r \vec{H}(\vec{r},t) + \mu_0 \vec{M}_0(\vec{r},t)$$

$\boldsymbol{H}$	: magnetic field strength	[A/m]
$\boldsymbol{B}$	: magnetic flux density	[T]
$\boldsymbol{M_0}$	: remanent magnetization	[A/m]
$\boldsymbol{\mu_0}$	: (magnetic) permeability of vacuum	[H/m]
	• value: $\mu_0 = \mathbf{4\pi \cdot 10^{-7}}$ H/m	
$\boldsymbol{\mu_r}$	: relative (magnetic) permeability	[-]

Electric

$$\vec{D}(\vec{r},t) = \varepsilon_0 \varepsilon_r \vec{E}(\vec{r},t) + \vec{P}(\vec{r},t)$$

Out of scope in 5LWF0!

$\boldsymbol{E}$	: electric field strength	[V/m]
$\boldsymbol{D}$	: electric flux density	[C/m^2]
$\boldsymbol{P}$	: polarization	[C/m^2]
$\boldsymbol{\varepsilon_0}$	: (electric) permittivity of vacuum	[F/m]
	• value†: $\varepsilon_0 = \mu_0^{-1} c^{-2} = \mathbf{8{,}854 \cdot 10^{-12}}$ F/m	
$\boldsymbol{\varepsilon_r}$	: relative (electric) permittivity	[-]

† c denotes the speed of light in vacuum, i.e. $c = 2.998 \cdot 10^8$ m/s

Constitutive relations (material properties)

- ***Hard*-magnetic materials: $M_0 \neq 0$**
  - Assumed to be **linear** when utilized properly! (In later lectures more)
- **Non-linear *soft*-magnetic materials: $M_0 = 0$ and $\mu_r\,(H)$**

Isotropic

Scalar *material properties that depend only on the local* ***magnitude*** *of the fields*

$$\mu_r(\vec{r}) = f_\mu\left(\left|\vec{H}(\vec{r})\right|\right)$$

$$\Downarrow$$

$$\vec{B}(\vec{r}) = \mu_0 f_\mu\left(\left|\vec{H}(\vec{r})\right|\right)\vec{H}(\vec{r})$$

Anisotropic

Tensor *material properties that depend on the local* ***spatial*** *components of the fields*

Out of scope

$$\vec{\mu}_r(\vec{r}) = \vec{f}_\mu\left(\vec{H}(\vec{r})\right)$$

in 5LWF0!

$$\Downarrow$$

$$\vec{B}(\vec{r}) = \mu_0 \vec{f}_\mu\left(\vec{H}(\vec{r})\right)\cdot\vec{H}(\vec{r})$$

The magnetic vector potential (MVP) for 2D problems

By introducing the MVP the number of PDEs can be reduced

- Gauss's law for magnetism: $\nabla \cdot \vec{B} = 0$
- Applying the vector identity $\nabla \cdot (\nabla \times \vec{A}) = 0$. Hence,

$$\boxed{\vec{B} = \nabla \times \vec{A}}$$

$$\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$$
$$\nabla \cdot \left(\nabla \times \vec{A}\right) = 0$$
$$\nabla \times (\nabla \varphi) = 0$$

- Reduction of a **system** of coupled PDEs to a single PDE (where reluctivity $\nu = \frac{1}{\mu}$).

2D elliptic PDE →

$$\boxed{\nabla \cdot [\nu(x,y)\nabla A_z(x,y,t)] = -J_z(x,y,t)}$$

Poisson equation for linear materials: constant reluctivity →

$$\boxed{\nu \nabla^2 A_z(x,y,t) = -J_z(x,y,t)}$$

Vector potential in 2D problems

In 2D problems the **flux density field** is confined to a single plane ➔ The **MVP** has only one nonzero spatial component as result of the curl operation

2D Cartesian	***B*** confined to the ***xy***-plane	$A_x(x,y,t) = A_y(x,y,t) = 0$	$A_z(x,y,t) \neq 0$
2D polar	***B*** confined to the ***rθ***-plane	$A_r(r,\theta,t) = A_\theta(r,\theta,t) = 0$	$A_z(r,\theta,t) \neq 0$
2D cylindrical	***B*** confined to the ***rz***-plane	$A_r(r,z,t) = A_z(r,z,t) = 0$	$A_\theta(r,z,t) \neq 0$

Additional source terms

Decomposition of the current density source term, $J_z = J_{\text{free}} + J_\sigma + J_M$:

- **Free-current term only:** an independently imposed current source

$$J_z(x, y, t) = J_{\text{free}}(x, y, t)$$

- **Eddy-current term only:** current flow in (homogeneous) conductive media (σ>0) when exposed to EMF inducing time-varying magnetic fields (Faraday's law)

$$J_z(x, y, t) = J_\sigma(x, y, t) = \sigma \vec{E}_z(x, y, t) = -\sigma \frac{\partial A_z(x,y,t)}{\partial t}$$

- **Permanent magnet term only:** hard-magnetic material ($J_{\text{free}} = 0$, $M \neq 0$)

$$0 = \nabla \times \vec{H}(x, y) = \nabla \times \left(\nu_0 \nu_{\text{r}}(x, y) \left(\vec{B}(x, y) - \mu_0 \vec{M}(x, y) \right) \right) \Rightarrow$$

$$J_z(x, y) = \nabla \times \left(\nu_{\text{r}}(x, y) \vec{M}(x, y) \right)$$

General PDE for additional source terms (parabolic PDE)

PDE for 2D electromechanical, **non-linear** problems:

$$\left(\nabla \cdot (\nu(x,y)\nabla) - \sigma \frac{\mathrm{d}}{\mathrm{d}t}\right) A_z(x,y,t) = -J_z(x,y,t) - \nabla \times \left(\nu_r(x,y)\vec{M}_0(x,y)\right)$$

- Depending on the physical nature of the problem (or assumptions) terms might vanish
- In general, problems consist of a **combination** of **multiple** domains of **different** physical nature
- The magnetization term is the only source term that is not defined in the *z*-direction.
  - Curl operation results in a vector in the z-direction
- For most practical cases, the PDE cannot be solved analytically, not even for linear materials
- *What is still missing to uniquely solve this parabolic PDE?*

Electromagnetic Boundary Conditions (BC)

- **Normal BC**
  - Equality of **flux density *normal*** to the boundary, Γ

$$\vec{n} \cdot \left(\vec{B}_2 - \vec{B}_1 \right) = 0$$

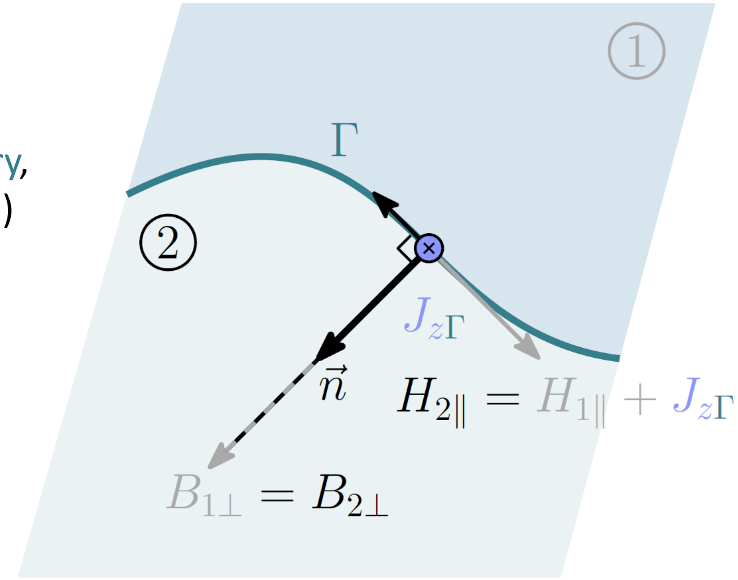
- **Tangential BC**
  - Jump of the **magnetic field strength *tangential*** the boundary, Γ, equal to the **surface current density** on it (in the z-direction)

$$\vec{n} \times \left(\vec{H}_2 - \vec{H}_1 \right) = \vec{J}_\Gamma$$

$$\vec{n} \times \left(\mu_1 \vec{B}_2 - \mu_2 \vec{B}_1 \right) = \mu_1 \mu_2 \vec{J}_\Gamma$$

- **At boundary of the (truncated) problem-space**
  - The field component exterior to problem space vanishes (=0)

Convention:
normal vector points from 1 into 2



Reduced formulations of the PDE

Transient: full PDE solved by time stepping (slow)

Nonlinear materials, Faraday: time-varying sources + movement, PMs can be modeled

$$\left(\nabla \cdot (\nu(x,y)\nabla) - \sigma \frac{\mathrm{d}}{\mathrm{d}t}\right) A_z(x,y,t) = -J_z(x,y,t) - \nabla \times \left(\nu_r(x,y)\vec{M}_0(x,y)\right)$$

Magnetostatic: time-invariant field quantities and sources (fast)

Nonlinear materials, **no** Faraday: DC sources**, no** movement, PMs can be modeled

$$\nabla \cdot (\nu(x,y)\nabla)\, A_z(x,y) = -J_z(x,y) - \nabla \times \left(\nu_r(x,y)\vec{M}_0(x,y)\right)$$

Steady-state AC: frequency domain: complex-valued field quantities and sources (fast)

Linear materials, Faraday: single harmonic AC sources**, no** movement, **no** PMs (*why?*)

$$\left(\nabla \cdot [\nu(x,y)\nabla] - j\omega\sigma\right) \bar{A}_z(x,y) = -\bar{J}_z(x,y)$$

Summary on Maxwell's equations and the PDE formulation

Basic electromagnetic principles of previous slides should already be (or become) ready knowledge

- Quasi-static Maxwell's equation
  - Interpretation of field distribution images (**no derivation of equations!**)
  - Insight in basic principles of operation of electromechanical devices
- Boundary conditions
  - Placing, assigning and **exploiting** boundary conditions properly
- Formulation of the governing partial differential equation
  - Selection of the proper solver type in FLUX2D, i.e. advantages vs. limitations
  - Impact of material assumptions on performance of electromechanical problems
  - Optimize 'cost' of simulation effort